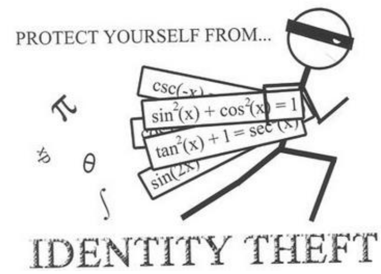
Welcome to AP Calculus AB

SUMMER REVIEW PACKET



You are receiving this packet because you are currently enrolled in AP Calculus AB for the 2019-2020 school year with Mrs. Wood. The purpose of this AP summer assignment is to review some pre-requisite math topics so we can jump right into learning new material when school starts in the fall. Throughout the entirety of our study of Calculus, it will be assumed that you have a thorough understanding of the topics contained in this packet.

Expectations:

- You are expected to have ALL of the problems included in this packet **finished and ready to hand in on the first day of class.** Please attempt every problem. I don't mind wrong answers but I will not accept question marks.
- No late submissions will be accepted.
- This assignment will be counted towards the homework portion of your grade for term 1. Assignments will be checked for work shown and effort put forth.
- You are welcome to collaborate with other students, but copying is not permitted.
- Questions about this packet will be answered during the first week of school and there will be a test over the material the second week of school.

If you have any difficulty with this packet during the summer, you might want to check out the following websites:

https://www.mathsisfun.com/algebra/
https://www.khanacademy.org/math/algebra-home
http://www.purplemath.com/modules/

OR you can email me at katherine.wood@braintreeschools.org (just know that you may not get an immediate repsonse).

Prerequisites for Calculus success:

- A good (if not thorough) understanding of precalculus concepts
- Excellent Algebra skills
- Working knowledge of the graphing calculator (you will need access to one on a daily basis)
- MOTIVATION, coupled with self-discipline and perseverance
- The ability to "dig your heels in" when the going gets tough
- TIME availability
- COMMITMENT

I look forward to an exciting and challenging year of Calculus!

Rules of exponents

Product of Powers Property $a^m \bullet a^n = a^{m+n}$	Power of a Power Property $(a^m)^n = a^{mn}$	Fractional Exponents $x^{\frac{m}{n}} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$
Negative Exponent Property $a^{-m} = \dfrac{1}{a^m}$ $a \neq 0$	Quotient of Powers Property $\dfrac{a^m}{a^n} = a^{m-n}$ $a \neq 0$	

Negative Exponents. Simplify each expression

a) $x^{-2}y^3$ b) $\dfrac{x^2}{y^{-5}}$ c) $\dfrac{x^{-3}}{y^{-9}}$

Multiplying with exponents. Simplify without the use of a calculator.

a) $x^2y^3 \cdot x^{-7}y^5$ b) $x^{\frac{3}{2}} \cdot x^{\frac{1}{4}}$ c) $x^{-\frac{2}{3}} \cdot x^{\frac{2}{5}}$

Fractional Exponents. Evaluate each WITHOUT the use of a calculator.

a) $(27)^{\frac{4}{3}}$ b) $(25)^{\frac{3}{2}}$ c) $(8)^{-\frac{5}{3}}$ d) $(9)^{-\frac{3}{2}}$

Linear Functions

Writing equations of lines

Slope intercept form: $y = mx + b$	**Vertical line:** x = c (slope is undefined)
Point-slope form: $y - y_1 = m(x - x_1)$	**Horizontal line:** y = c (slope is 0)

a) Use slope-intercept form to find the equation of the line having a slope of 3 and a y-intercept of 5.

b) Determine the equation of a line passing through the point $(5, -3)$ with an undefined slope.

c) Determine the equation of a line passing through the point $(-4, 2)$ with a slope of 0.

d) Use point-slope form to find the equation of a line passing through the point $(6, 5)$ with a slope of $\frac{2}{3}$.

e) Find the equation of a line passing through the point $(2, 8)$ and parallel to the line $y = \frac{5}{6}x - 1$.

f) Find the equation of a line perpendicular to the y-axis passing through the point $(4, 7)$.

g) Find the equation of a line passing through the points $(-3, 6)$ and $(1, 2)$.

h) Find the equation of the line that is perpendicular to the line $2x + 3y - 8 = 0$ at the point $(1, 2)$.

i) The line with slope 5 that passes through the point $(-1, 3)$ intersects the x axis at a point. What are the coordinates of this point?

Functions:

Domain/Range- Find the domain and range of the following functions. You should be able to complete **WITHOUT** the aid of a calculator. Leave answers in interval notation. ($3 < x \leq 6$ is written as $(3, 6]$ and $x > 3$ is written as $(3, \infty)$)

a) $f(x) = x^2 - 5$

b) $f(x) = -\sqrt{x + 3}$

c) $f(x) = 3sinx$

d) $f(x) = \frac{2}{x-1}$

e) $f(x) = |x - 3|$

f) $f(x) = x^3 + 3x + 1$

Function Operations-

To evaluate a function for a given value, simply plug the value into the function for x.

Recall: $(f \circ g)(x) = f(g(x))$ OR $f[g(x)]$ read "*f* of *g* of ***x***" Means to plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x) = 2x^2 + 1$ *and* $g(x) = x - 4$ find *f(g(x))*.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

a) Given $f(x) = 2x^2 + 4x$ and $g(x) = x - 3$, find each:

i) $(f + g)(x)$

ii) $(fg)(x)$

iii) $f(g(2))$

iv) $g(f(2))$

b) Given the table of values, find each:

x	$f(x)$	$g(x)$	$h(x)$
2	3	5	8
5	1	3	1
8	7	2	9

i) $(f - h)(8)$

ii) $f(g(2))$

iii) $f(g(h(2))$

iv) $\left(\frac{h}{f}\right)(2)$

c) Find $\frac{f(x+h)-f(x)}{h}$ for the given function f:

i) $f(x) = 9x + 3$

ii) $f(x) = x^2 - 1$

Polynomials

<u>Factoring</u> - Factor each of the following completely.

a) $a^2 - b^2$

b) $a^3 - b^3$

c) $8x^3 + y^3$

d) $4x^2 - 21x - 18$

e) $2x^2 + x - 3$

f) $3x^2 + 6x^3 - 9x$

FACTOR----DO NOT DISTRIBUTE FIRST

g) $(x+1)^3(4x-9) - (16x+9)(x+1)^2$

h) $(x-1)^3(2x-3) - (2x+12)(x-1)^2$

Solving quadratics – Find the value(s) of x. Round to three decimals places if necessary.

a) $x^2 + 11x + 18 = 0$

b) $x^2 - 5x + 1 = 7$

c) $7x^2 - 9x + 1 = 0$

d) $(x + 2)^2 = 36$

e) $x^2 - 10x + 25 = 0$

f) $2x^2 + 7x - 4 = 0$

Rational Functions

Multiplying and Dividing- Multiply or divide as indicated. Express answer in lowest terms.

a) $\frac{y^2 - 2y}{15} \cdot \frac{5}{y-2}$

b) $\frac{3y^2 - 12}{y^2 + 4y + 4} \div \frac{y^3 - 2y^2}{y^2 + 2y}$

c) $\frac{x^2 - 2x}{x^2 + 6x + 9} \div \left(\frac{x^2 - 4}{x^2 + 3x} \cdot \frac{x}{x+2} \right)$

d) $\frac{y^2 + y - 12}{y^2 + y - 30} \cdot \frac{y^2 + 5y + 6}{y^2 - 2y - 3} \cdot \frac{y^2 + 7y + 6}{y+3}$

e) $\frac{(x+1)^3(4x-9)-(16x+9)(x+1)^2}{(x-6)(x+1)^3}$

f) $\frac{3x(x+1)-2(2x+1)}{(x-1)^2}$

Complex Fractions- Simplify each complex fraction:

Example:

$$\frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} = \frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} \cdot \frac{x(x-4)}{x(x-4)} = \frac{-2(x-4)+3x(x)}{5(x)(x-4)-1(x)} = \frac{-2x+8+3x^2}{5x^2-20x-x} = \frac{3x^2-2x+8}{5x^2-21x}$$

a) $\dfrac{\frac{3}{5}-\frac{2}{3}}{\frac{2}{3}-\frac{5}{6}}$

b) $\dfrac{\frac{2}{w}-\frac{3}{w^2}}{2-\frac{3}{w}}$

c) $\dfrac{4-7y^{-1}}{3-2y^{-1}}$

d) $\dfrac{\frac{3}{x^2-1}+2}{\frac{5}{x+1}-\frac{4}{x-1}}$

e) $\dfrac{\frac{1}{2}x^2+1}{\frac{4}{x}}$

f) $\dfrac{2-\frac{1}{x}}{\frac{1}{x^2}-\frac{1}{2}}$

Hint for parts g) and h) – Rewrite negative exponents as fractions.

g) $\dfrac{(x+1)^{-\frac{1}{2}}+3x(x+1)^{\frac{1}{2}}}{4x^2}$

h) $\dfrac{\frac{1}{2}(2x-5)^{\frac{1}{3}}-3x(2x-5)^{-\frac{2}{3}}}{5x}$

Finding Vertical and Horizontal Asymptotes.

Vertical Asymptotes: Reduce fraction and set denominator equal to zero.

Horizontal Asymptotes:

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is $y = 0$.

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

a) Find the vertical and horizontal asymptotes of each function:

i) $f(x) = \frac{3x}{x^2-3x+2}$

ii) $f(x) = \frac{1}{x^2}$

iii) $\frac{x^3-2x+1}{x^2+1}$

iv) $f(x) = \frac{x^2+9x-22}{4x^2-16}$

v) $f(x) = \frac{x}{\sqrt{3x^2-1}}$

vi) $f(x) = \frac{4x^2-4}{3x^2-6x+3}$

Intercepts and Points of Intersection

Intercepts:

To find the x-intercepts, let $y = 0$ in your equation and solve.
To find the y-intercepts, let $x = 0$ in your equation and solve.

Example: $y = x^2 - 2x - 3$

$x-$int. $(Let\ y = 0)$

$0 = x^2 - 2x - 3$

$0 = (x-3)(x+1)$

$x = -1\ or\ x = 3$

$x-$intercepts $(-1,0)\ and\ (3,0)$

$y-$int. $(Let\ x = 0)$

$y = 0^2 - 2(0) - 3$

$y = -3$

$y-$intercept $(0,-3)$

a) For each function, find the x and y intercepts by hand.

i) $y = 2x - 5$

ii) $y = x^2 + x - 2$

b) Use your graphing calculator to find the x intercept(s) of each of the following functions. Round to 3 decimals places.

i) $y = x^3 - 2x - 5$

ii) $f(x) = x^4 - 3x^3 + 2$

Points of Intersection

Use substitution or elimination method to solve the system of equations.
Example:

$$x^2 + y - 16x + 39 = 0$$
$$x^2 - y^2 - 9 = 0$$

Elimination Method

$2x^2 - 16x + 30 = 0$

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3$ *and* $x = 5$

Plug $x=3$ *and* $x = 5$ into one original

$3^2 - y^2 - 9 = 0$ $\quad$ $5^2 - y^2 - 9 = 0$

$-y^2 = 0$ $\quad$ $16 = y^2$

$y = 0$ $\quad$ $y = \pm 4$

Points of Intersection $(5,4)$, $(5,-4)$ *and* $(3,0)$

Substitution Method

Solve one equation for one variable.

$y^2 = -x^2 + 16x - 39$ (1st equation solved for y)

$x^2 - (-x^2 + 16x - 39) - 9 = 0$ Plug what y^2 is equal to into second equation.

$2x^2 - 16x + 30 = 0$ *(The rest is the same as previous example)*

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3$ *or* $x - 5$

a) For each pair of functions, find the point(s) of intersection by hand.

i) $x + y = 8$

$4x - y = 7$

ii) $x^2 + y = 6$

$x + y = 4$

b) For each pair of functions, find the point(s) of intersections using your graphing calculator. Hint: Reduce to one equation in terms of one variable, solve for 0, put into y = , and find roots on calculator. Round to 3 decimals places.

i) $y = sinx$

$2x - y = 6$

ii) $3x + y = 10$

$x^2 + 2x = y^2 + 4$

Trigonometry

Radian and Degree Measure. FYI, in Calculus, we will ONLY use radian measure

Use $\frac{180^\circ}{\pi\, radians}$ to get rid of radians and convert to degrees.	Use $\frac{\pi\, radians}{180^\circ}$ to get rid of degrees and convert to radians.

a) Covert to degrees:

i) $\frac{5\pi}{6}$

ii) $\frac{4\pi}{5}$

iii) 2.63 radians (round to 3 decimals places)

b) Convert to radians: (leave as reduced fractions)

i) 45°

ii) 237°

Evaluating all 6 trig functions. You may not use a calculator to evaluate the following. These must be done by the unit circle (memorized) or by the reference triangle method shown above.

a) $\sin\left(\frac{3}{4}\pi\right)$

b) $\cos\left(-\frac{3}{4}\pi\right)$

c) $\cot\left(-\frac{13}{6}\pi\right)$

d) $\csc \pi =$

e) $\tan\left(\frac{4\pi}{3}\right)$

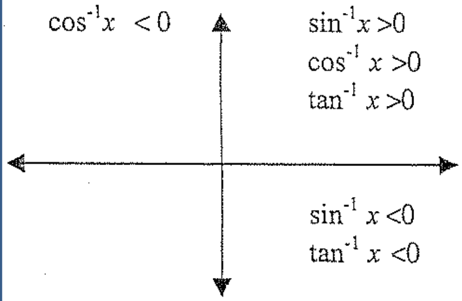
f) $\sec\left(\frac{5\pi}{6}\right)$

g) $\cos(0)$

h) $\sec\left(\frac{3\pi}{2}\right)$

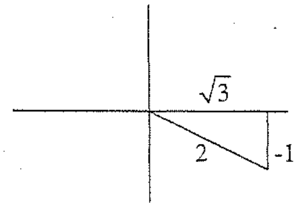
i) $tan\left(\frac{\pi}{2}\right)$

Inverse Trig



Example:
Express the value of "y" in radians.

$y = \arctan\frac{-1}{\sqrt{3}}$ Draw a reference triangle.



This means the reference angle is 30° or $\frac{\pi}{6}$. So, $y = -\frac{\pi}{6}$ so that it falls in the interval from $\frac{-\pi}{2} < y < \frac{\pi}{2}$

Answer: $y = -\frac{\pi}{6}$

Without using a calculator, give an exact value for θ. Be sure to use the correct **domain**.

a. $\theta = \sin^{-1}\frac{1}{2}$

b. $\theta = \tan^{-1}\frac{\sqrt{3}}{3}$

c. $\theta = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

d. $\theta = \sin^{-1}1$

e. $\theta = \sin^{-1}\left(-\frac{1}{2}\right)$

f. $\theta = \cos^{-1}1$

g. $\theta = \cos^{-1}\frac{\sqrt{3}}{2}$

h. $\theta = \sin^{-1}(-1)$

i. $\theta = \tan^{-1}(-1)$

j. $\theta = \cos^{-1}\left(-\frac{1}{2}\right)$

k. $\theta = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

l. $\theta = \tan^{-1}(-\sqrt{3})$

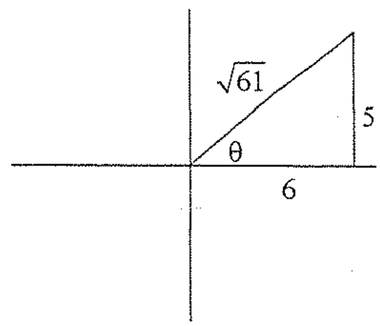
Example: Find the value without a calculator.

$$\cos\left(\arctan\frac{5}{6}\right)$$

Draw the reference triangle in the correct quadrant first.

Find the missing side using Pythagorean Thm.

Find the ratio of the cosine of the reference triangle.



$$\cos\theta = \frac{6}{\sqrt{61}}$$

Find each of the following without the use of a calculator:

a) $\tan\left(arccos\left(\frac{2}{3}\right)\right)$

b) $\sec\left(sin^{-1}\left(\frac{12}{13}\right)\right)$

c) $\sin\left(arctan\left(\frac{12}{5}\right)\right)$

d) $\cos\left(arcsin\left(\frac{7}{8}\right)\right)$

Inverses. Use the functions $f(x)$ and $g(x)$ below to determine the following:

x	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	2	-2	-4	3	-3	-1	0	4	3

x	-4	-3	-2	-1	0	1	2	3	4
$g(x)$	1	2	3	4	-3	-2	-4	-1	0

a) $f^{-1}(0) =$

b) $g^{-1}(f(4)) =$

c) $(g + f^{-1})(-4)$

Exponential and Logarithmic Functions

Definition

$y = \log_b x$ is equivalent to $x = b^y$

Example

$\log_5 125 = 3$ because $5^3 = 125$

Special Logarithms

$\ln x = \log_e x$ natural log

$\log x = \log_{10} x$ common log

where $e = 2.718281828\ldots$

Logarithm Properties

$\log_b b = 1$ $\quad \log_b 1 = 0$

$\log_b b^x = x$ $\quad b^{\log_b x} = x$

$\log_b (x^r) = r \log_b x$

$\log_b (xy) = \log_b x + \log_b y$

$\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$

The domain of $\log_b x$ is $x > 0$

$2^x = 7$

Log both sides

$\text{Log}(2^x) = \text{Log}(7)$

Log rule: $\text{Log}(x)^n = n\text{Log}(x)$

$x\text{Log}(2) = \text{Log}(7)$

Rearrange & Solve

$x = \frac{\text{Log}(7)}{\text{Log}(2)} = 2.807$

Solving exponential Equations (No Logs). Solve each equation without the aid of a calculator.

a) $3^x = 9^{x^2}$ b) $\frac{27^{2x-5}}{9^{3x+1}} = 3^4$ c) $4^{2-3x} = \frac{1}{2}$ d) $e^{x^2} = e^{3x} \cdot \frac{1}{e^2}$

Evaluating Logs. Simplify each of the following without a calculator:

a) $log_6 1$ b) $ln e^{10}$ c) $log_{\frac{1}{5}} 25$ d) $log_{\sqrt{2}} 8$

Log Properties. Express each of the given logarithms as the sum and/or difference of simpler logs, without any radicals or exponents:

a) $log(1000ab^3c)$ b) $ln\left(\frac{\sqrt{x}}{y^2}\right)$ c) $log_4\left(\frac{2x\sqrt{y}}{\sqrt[3]{z+1}}\right)$

Express each as a single logarithm:

a) $3 + log_2 x - 9 log_2 y$

b) $7lnx + 2lny - 1 + \frac{1}{3}\ln(z - 7)$

<u>Log Equations.</u> Solve each equation WITHOUT the use of a calculator:

a) $log_6(9x + 4) = log_6 19$

b) $logx - 4log5 = -2$

c) $\ln(x - 1) - \ln(x + 1) = -2$

d) $2log_4 x - log_4(x - 1) = 1$

Solve each of the given equations with a calculator, correct to the nearest 3 decimal places:

e) $6^x = 2.4$

f) $15(1.08)^{x-1} = 400$

g) $4^x = 7^{x-1}$

h) $4\ln(2x + 3) = 11$

i) $2log_4(x + 1) =$